Midterm **Name:** ____________________

Sample

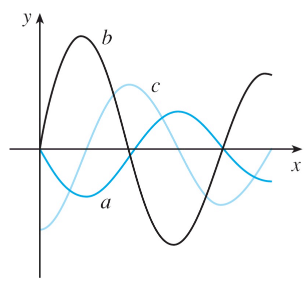
Show your work! You are allowed to use a 4 × 6 card with notes written on one side and a dedicated calculator.

1. Consider the function $f(x) = x^2 - x$ on the interval $0 \leq x \leq 2$.

   (a) Set up the Riemann sum to approximate the area under the curve. Use $n = 4$ subintervals and select x_i^* to be the right end point of each subinterval.

   (b) Use the Fundamental Theorem of Calculus to get the exact value for the integral.

2. The following figure shows the graphs of f, f', and $\int_0^x f(t)dt$. Identify each graph.



3. Evaluate the following integrals.

   (a) $\displaystyle\int_1^2 (8x^3 + 3x^2)\ dx$

   (b) $\displaystyle\int_1^9 \frac{\sqrt{u} - 2u^2}{u}\ du$

   (c) $\displaystyle\int_0^1 v^2 cos(v^3)\ dv$

   (d) $\displaystyle\int \sin \pi t \cos \pi t\ dt$

   (e) $\displaystyle\int_0^4 |3 - x|\ dx$

   (f) $\displaystyle\int_0^4 \frac{1}{16 + t^2}\ dt$

   (g) $\displaystyle\int \frac{x}{\sqrt{1 - x^4}}\ dx$

   (h) $\displaystyle\int \frac{x + 1}{x^2 + 2x}\ dx$

4. Find the derivative of the following function.

$$F(x) = \int_1^x \sqrt{1+t^4}\, dt$$

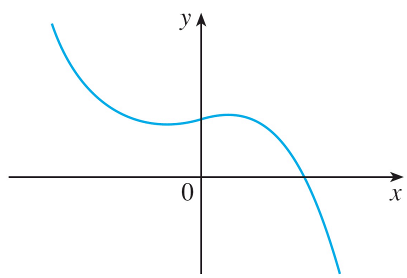
5. A particle moves along a line with velocity function $v(t) = t^2 - t$, where v is measured in meters per second.

   (a) Find the displacement of the particle during the time interval $[0, 5]$.

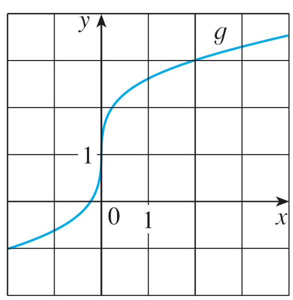
   (b) Find the total distance travelled by the particle during the time interval $[0, 5]$.

6. Find the average value of the function $f(x) = x\sin(x^2)$ on the interval $[0, 2]$.

7. The graph of f is shown. Is f one-to-one? Explain your reasoning.



8. The graph of g is given.



   (a) Estimate the value of $g^{-1}(2)$.

   (b) Sketch the graph if g^{-1}

9. Find the inverse function $f(x) = \dfrac{x+1}{2x+1}$.

10. Find the exact value of each expression.

   (a) $e^{2\ln 3}$

   (b) $\ln e^{\pi}$

   (c) $\log_{10} 25 + \log_{10} 4$

11. Solve the following equations for x.

    (a) $\ln(x+1) + \ln(x-1) = 1$

    (b) $\log_5(c^x) = d$

12. Differentiate the following functions

    (a) $y = \ln(x \ln x)$

    (b) $y = \sqrt{\tan^{-1} x}$

    (c) $f(x) = 4^{2x}$

    (d) $g(x) = \sin^{-1}(4x)$

13. A bacteria culture contains 200 cells initially and grows at a rate proportional to its size. After half an hour the population has increased to 360 cells.

    (a) Find the number of bacteria as a function of time t measured in hours.

    (b) Find the number of bacteria after 4 hours.

    (c) Find the rate of growth after 4 hours.

    (d) When will the population reach 10,000?

14. Simplify the expression.

    (a) $\sin(\tan^{-1} x)$

    (b) $\cos(2 \tan^{-1} x)$

Table of Integrals and Derivatives

(1) $\displaystyle\int \frac{1}{u}\ du = \ln|u| + C$

(2) $\displaystyle\int \frac{1}{a^2+u^2}\ du = \frac{1}{a}\tan^{-1}\frac{u}{a} + C$

(3) $\displaystyle\int \frac{1}{\sqrt{a^2-u^2}}\ du = \sin^{-1}\frac{u}{a} + C$

(4) $\displaystyle\int \frac{1}{u\sqrt{u^2-a}}\ du = \frac{1}{a}\sec^{-1}\left|\frac{u}{a}\right| + C$

(5) $\displaystyle\int a^u\ du = \frac{a^u}{\ln a} + C, a \neq 1$

(6) $\displaystyle\int \ln u\ du = u\ln u - u + C$

(7) $\displaystyle\int \tan u\ du = \ln|\sec u| + C$

(8) $\displaystyle\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$

(9) $\displaystyle\frac{d}{dx}(\cos^{-1}x) = \frac{-1}{\sqrt{1-x^2}}$

(10) $\displaystyle\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$

(11) $\displaystyle\frac{d}{dx}(\cot^{-1}x) = \frac{-1}{1+x^2}$

(12) $\displaystyle\frac{d}{dx}(\sec^{-1}x) = \frac{1}{|x|\sqrt{x^2-1}}$

(13) $\displaystyle\frac{d}{dx}(\csc^{-1}x) = \frac{-1}{|x|\sqrt{x^2-1}}$

(14) $\displaystyle\frac{d}{dx}(\log_a x) = \frac{1}{(\ln a)x}$

(15) $\displaystyle\frac{d}{dx}(a^x) = (\ln a)a^x$